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Effective Mass Matrix for Light Neutrinos Consistent with Solar and Atmospheric Neutrino Experiments

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Abstract

We propose an effective mass matrix for light neutrinos which is consistent with the mixing pattern indicated by solar and atmospheric neutrino experiments. Two scenarios for the mass eigenvalues are discussed and the connection with double beta decay is noted.

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There are significant hints for neutrino mass coming from solar [1] and atmospheric [2] neutrino experiments and they present an interesting theoretical challenge. Solar neutrino experiments suggest that electron neutrinos oscillate into other lepton flavors with a small mixing angle [3], $\sin^2 2\theta \approx 7\times10^{-3}$, and a small mass-squared difference, $\Delta m^2 \approx 5\times10^{-6}$ eV^2. Atmospheric neutrino experiments, on the other hand, suggest that muon neutrinos oscillate into tau neutrinos with maximal mixing [4], $\sin^2 2\theta = 1$, and a much larger mass-squared difference, $\Delta m^2 \approx 2 \times 10^{-2}$ eV^2. How can we accommodated these very different sets of oscillation parameters within one neutrino mass matrix?

Here we wish to propose an effective mass matrix, $\mathcal{M}$, of the form:

$$\bar{\nu}\mathcal{M}\nu \equiv (\,\bar{\nu}_e, \quad \bar{\nu}_\mu, \quad \bar{\nu}_\tau\,) \begin{pmatrix} m_1 & m_2 & m_2 \\ m_2 & M_1 & -M_2 \\ m_2 & -M_2 & M_1 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \tag{1}$$

which possesses the desired properties. The matrix is real and symmetric and thus conserves CP irrespective of whether it originates from Dirac or Majorana mass terms. Because the diagonal elements in the μ-τ sector are equal to one another, there will be maximal mixing between these two flavors; furthermore, if m_2 is taken to be much smaller than M_1 and M_2, then mixing with the electron flavor will be weak. The same element m_2 is used in the e-μ and e-τ positions so that matrix can be diagonalised in a simple two-step process and the electron neutrino can be decoupled from the heaviest mass eigenvector. The four constants in the matrix can then be chosen to fit various scenarios consistent with the oscillation hints. We shall consider two of them below.

The first step in diagonalising the mass matrix is a rotation of 45° in the μ-τ sector:

$$U_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{2}$$

$$\tilde{U}_1\mathcal{M}U_1 = \begin{pmatrix} m_1 & \sqrt{2}m_2 & 0 \\ \sqrt{2}m_2 & M_1 - M_2 & 0 \\ 0 & 0 & M_1 + M_2 \end{pmatrix}. \tag{3}$$

The second is to rotate the upper 2×2 submatrix through an angle θ:

$$U_2 = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{4}$$

where

$$\tan 2\theta = -\frac{2\sqrt{2}m_2}{M_1 - M_2 - m_1}. \tag{5}$$

In the limit that m_2 is much smaller than the diagonal elements of $\mathcal{M}$, the eigenvalues can be expressed in terms of a small parameter Δ,

$$M_x = m_1 - \Delta, \tag{6}$$

$$M_y = M_1 - M_2 + \Delta, \tag{7}$$

$$M_z = M_1 + M_2 \tag{8}$$

$$\Delta = \frac{2(m_2)^2}{M_1 - M_2 - m_1}, \tag{9}$$

and

$$\tilde{U}_2\tilde{U}_1\mathcal{M}U_1U_2 = \begin{pmatrix} M_x & 0 & 0 \\ 0 & M_y & 0 \\ 0 & 0 & M_z \end{pmatrix}. \tag{10}$$

In terms of the mass eigenstates ν_w ($w = x, y, z$), the flavor eigenstates become:

$$\begin{aligned} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} &= U_1U_2 \begin{pmatrix} \nu_x \\ \nu_y \\ \nu_z \end{pmatrix} \\ &= \begin{pmatrix} 1 & \frac{\Delta}{\sqrt{2}m_2} & 0 \\ -\frac{\Delta}{2m_2} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{\Delta}{2m_2} & \frac{1}{\sqrt{2}} & +\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu_x \\ \nu_y \\ \nu_z \end{pmatrix} \end{aligned} \tag{11}$$

It follows from this expression that ν_e has no coupling to the heaviest mass eigenvector ν_z, and that it oscillates into a coherent combination

$$\nu_a = \frac{1}{\sqrt{2}}(\nu_\mu + \nu_\tau) \tag{12}$$

of muon and tau neutrinos with mixing angle

$$\sin\theta_{ea} \approx \frac{\Delta}{\sqrt{2}m_2} = \frac{\sqrt{2}m_2}{M_1 - M_2 - m_1}. \tag{13}$$

The solar neutrino data [1,3], $\sin^2 2\theta \approx 7 \times 10^{-3}$, then implies that

$$\frac{\Delta}{\sqrt{2}m_2} = \frac{\sqrt{2}m_2}{M_1 - M_2 - m_1} \approx \frac{1}{23}, \tag{14}$$

or

$$\frac{\Delta}{M_1 - M_2 - m_1} \approx \frac{1}{500}. \tag{15}$$

Having accommodated the mixing angles suggested by the solar and atmospheric data, we now turn to the mass eigenvalues (M_x, M_y, M_z). The information available to us concerns mass-squared differences, namely

$$\Delta_{yx} = (M_y)^2 - (M_x)^2 \approx 5 \times 10^{-6}\ \text{eV}^2 \tag{16}$$

from solar neutrino experiments [1,3], and

$$\Delta_{zy} = (M_z)^2 - (M_y)^2 \approx 2 \times 10^{-2}\ \text{eV}^2 \tag{17}$$

from atmospheric ones [4]. There is obviously a whole family of solutions to these equations and in order to extract interesting physics from them, we must make some assumption about the magnitudes of the masses in relation to the mass differences. Two extreme possibilities are that the $(M_w)^2$ are either of the same order as the Δ_{uv}, or much greater than them.

In the former case, the parameters of the original mass matrix $\mathcal{M}$ turn out to be:

$$\begin{aligned} m_1 &\ll 10^{-3} \text{ eV}, \\ m_2 &= 7 \times 10^{-5} \text{ eV}, \\ M_1 - M_2 &= 2 \times 10^{-3} \text{ eV}, \\ M_1 + M_2 &= 1.4 \times 10^{-1} \text{ eV}. \end{aligned} \tag{18}$$

In the latter case, recent cosmological arguments [5] indicate masses of order 2 eV; taking this as our guide, we then find that all three diagonal elements of $\mathcal{M}$ are close to 2 eV:

$$\begin{aligned} m_1 &= 2 \text{ eV} - \delta - \eta, \\ m_2 &= 4 \times 10^{-8} \text{ eV}, \\ M_1 - M_2 &= 2 \text{ eV} - \delta, \\ M_1 + M_2 &= 2 \text{ eV} + \delta, \\ \delta &= 2.5 \times 10^{-3} \text{ eV}, \\ \eta &= 1.3 \times 10^{-6} \text{ eV}. \end{aligned} \tag{19}$$

It is interesting to note some numerical relationships between the fine-tuned parameters of this form of $\mathcal{M}$. The parameter δ is the square of $\frac{\Delta}{\sqrt{2}m_2}$ and η is roughly the square of δ; in addition m_2 is close to the cube of δ. This suggests that the mass matrix might be a power series expansion of some relatively simple underlying matrix.

The closeness of m_1, the e-e element of $\mathcal{M}$, to 2 eV presents an interesting problem with regard to the Dirac versus Majorana nature of the mass matrix. If $\mathcal{M}$ is constructed from Majorana masses, then the amplitude for no-neutrino double beta decay will be proportional to m_1. Now the bounds on the effective Majorana mass for double beta decay are in the neighborhood of 1–2 eV [6]. Consequently we may be in a very interesting position with respect to this lepton number violating process: if the experimental sensitivity can be improved by an order of magnitude [7], and if our $(M_w)^2 \gg \Delta_{uv}$ version of the mass matrix is correct, then we should actually detect no-neutrino double beta decay.

Since m_1 is very small in the $(M_w)^2 \approx \Delta_{uv}$ version of $\mathcal{M}$, no-neutrino double beta decay would be effectively undetectable and it cannot be determined whether this version of the mass matrix is Majorana or Dirac. Thus a failure to detect the process would indicate that either the $(M_w)^2 \approx \Delta_{uv}$ mass matrix is correct, or that the $(M_w)^2 \gg \Delta_{uv}$ one must be a Dirac mass matrix.

We have not constructed a specific gauge theory model for $\mathcal{M}$, but we are confident that it is possible to do so. For example, Albright and Nandi [8] have developed a procedure for starting with low energy data and evolving SO(10) Grand Unified Theories at the unification scale; indeed one of their scenarios matches the mass eigenvalues of our $(M_w)^2 \approx \Delta_{uv}$ case and can probably be adapted to yield a similar mixing matrix. Similarly, Caldwell and Mohapatra [9], motivated by cosmological arguments, have considered a mass matrix similar to our $(M_w)^2 \gg \Delta_{uv}$ case in the context of SO(10) and left-right symmetric models. Other models have been based upon radiative corrections [10], SUSY [11], and even gravitation [12], but the corresponding mass matrices tend to have a different structure from the one proposed here.

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